

## THE TRACK FORMATION THEORY BY KATZ AND COWORKERS APPLIED TO IONIZATION MEASUREMENTS IN NUCLEAR EMULSION

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### RÉSUMÉ

On a comparé les profils théoriques et expérimentaux de traces d'ions lourds dans les émulsions nucléaires. Les expériences ont été réalisées avec des ions, de charge comprise entre 14 et 26 et de vitesse  $0,3 \leq \beta \leq 0,8$ , dont les profils de traces sont mesurés à l'aide de deux photomètres à diaphragme rectangulaire. Les calculs théoriques utilisent la théorie de Katz de distribution de l'énergie autour de la trace et la transmission de la lumière directe et diffusée par les grains de bromure d'argent.

L'accord entre données théoriques et expérimentales est excellent et on peut caractériser la réponse de chaque émulsion par une seule grandeur.

L'examen des données expérimentales d'autres auteurs a permis de constater que la théorie s'applique également pour les ions de  $Z > 26$ .

### ABSTRACT

A comparison was made of the theoretical and experimental profiles of heavy ions tracks in nuclear emulsions. The experiments were conducted with ions (charge interval  $14 \leq Z \leq 26$  and velocity interval  $0,3 \leq \beta \leq 0,8$ ) whose track profiles were measured by means of two photometers with rectangular slits. The theoretical calculations used Katz's theory on energy distribution around the track and transmittance of direct and diffused light by Ag-Br grains. There was a very good agreement between theoretical and experimental data, and a single quantity could be used to characterize an emulsion stack response. An examination of the experimental data obtained by other investigators showed that the theory also applied to ions with  $Z > 26$ .

The track formation theory by Katz and coworkers is based on the concept of the energy dose deposited by secondary electrons at different radial distances from the path of a passing ion. The theory was originally developed for calcu-

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lation of the track structure of heavy ions in nuclear emulsion [1, 2]. While the application of the theory in recent years has been extended to new areas of research, such as biology and medicine, the nuclear emulsion still seems to be best suited for a somewhat more detailed test of the basic elements of the theory. In this paper we give the results of such a test, based on a comparison of the theoretical and experimental data of the transmittance (or absorbance) of light in an emulsion volume containing the track.

The measurements have been made with two different types of nuclear track photometer. In both cases the cross-section of the beam of light is defined by rectangular slits which are adjusted so that their length dimension is parallel to the track. The basic difference between the photometers is in the width of the slit relative to that of the track, as shown by the sketch in Fig. 1.

The photometer of type 1 employs a slit whose image in the emulsion plane is much narrower than the core of the track. The transmittance is measured at different lateral distances from the track axis, yielding a transmittance profile of a short section of the track of the typical appearance shown in Fig 1.

The slit of a type 2 photometer is stationary and broader than the core of the track. To obtain the relevant background level, two further slits are

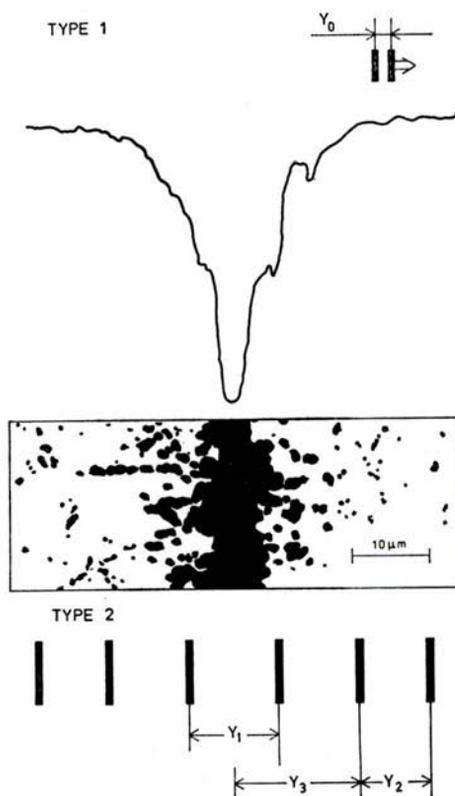


FIG. 1. — A sketch of the slit system and its orientation relative to the track for two types of photometer.

used, one on each side of the central slit which contains the track. By this photometer one records the mean transmittance in a short section of the track.

The theoretical calculation of transmittance is carried out in two steps. In the first step the distribution of grains at different radial distances  $t$  from the track axis is obtained. The second step deals with the transport of light through the emulsion at a given lateral distance  $y$  from the track axis.

To calculate the energy dose distribution  $E(t)$  one needs to know the cross-section for the production of secondary electrons of different energies, the angular distribution of the electrons, and their transport and energy dissipation in the emulsion. From  $E(t)$  we obtain the mean dose  $\bar{E}(t)$  deposited within the volume of a typical AgBr grain. The details of the calculation are discussed in a previous paper [3]. Within a limited interval of distances  $t$  we have the approximative relationship :

$$\bar{E}(t) \approx \frac{Z^2}{\beta^2 t^2}, \quad (1)$$

where  $Z$  is the charge number of the passing ion and  $\beta$  its velocity relative to that of light. From the mean dose we get the expected volume density  $\langle n(t) \rangle$  of developed grains as

$$\langle n(t) \rangle = n_0 [1 - \exp(-\bar{E}(t)/E_0)], \quad (2)$$

where  $n_0$  is the volume density of AgBr crystals in the undeveloped emulsion and  $E_0$  is the characteristic dose, i.e. the dose for rendering 63 p. cent of the crystals developable.

In the second step we use the grain distribution according to equation [2] to calculate the transmittance  $\tau_p(y)$  of a parallel beam of light,

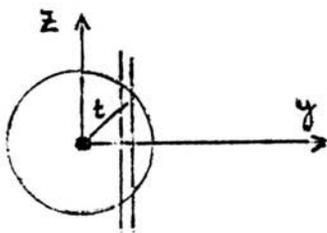
$$\tau_p(y) = e^{-\sigma N(y)}, \quad (3)$$

where  $\sigma$  is the cross-section for removal of light from the beam and  $N(y)$  is the projected number of grains per unit cross-sectional area of the beam at a lateral distance  $y$  from the track axis. The quantity  $N(y)$  is given by

$$N(y) = \int \langle n(t) \rangle dz, \quad (4)$$

$z$  being the height coordinate in the emulsion sheet, i.e.  $y^2 + z^2 = t^2$ , and the integration being carried out over the entire depth interval containing grains which belong to the track.

The photometers used in our measurements have a high aperture. Consequently we have to take into account the light which is



rescattered into the cone of acceptance. For a beam of parallel light this is a second order effect while in our apparatus it may reduce the absorptance of a track situated in the middle of the emulsion thickness by some 10 to 20 p. cent. The measured absorptance can conveniently be simulated by the expression

$$1 - \tau(y) = \sum_{v=0}^2 a_v [1 - \tau_p(y)]^v, \quad (5)$$

i.e. by a power series expansion in terms of the absorptance  $1 - \tau_p(y)$  of a parallel beam of light. Arguments for this choice and an attempt to a physical interpretation of the different terms in the expansion can be found in ref. [3].

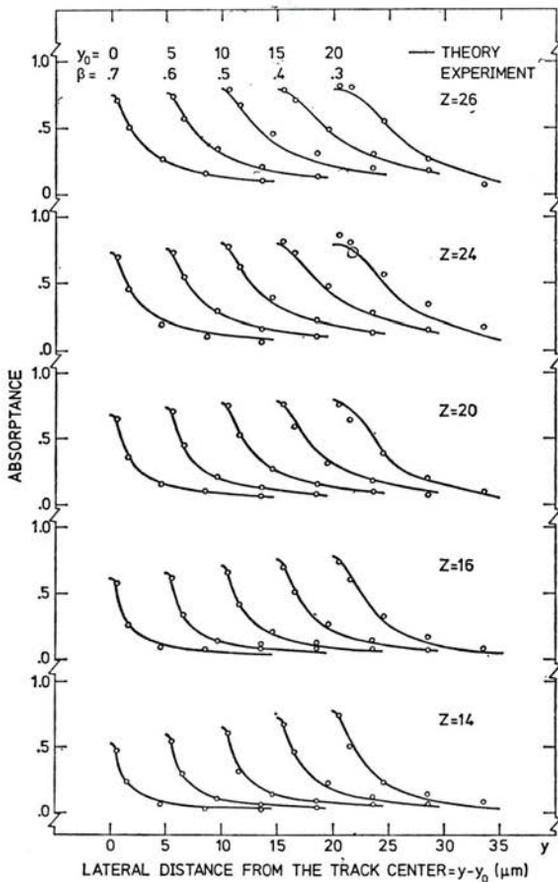


FIG. 2. — Absorptance as a function of lateral distance from track axis. Theoretical relationships shown by solid curves, experimental points by circles. Charge interval  $14 \leq Z \leq 26$ , velocity interval  $0.3 \leq \beta \leq 0.8$ .

In ref. [3] we have undertaken a comparison of experimental and theoretical absorptance profiles in the charge interval  $14 \leq Z \leq 26$  and the velocity interval  $0.3 \leq \beta \leq 0.8$ . The parameters needed for the calculation of absorptance, i.e. the cross-section  $\sigma$  and the coefficients  $a_p$  in equation (5), were determined from the experimental data, using a non-linear parameter optimization procedure. The main results of the comparison are shown in Fig. 2 which is a plot of absorptance as a function of lateral distance for ions with  $Z$  and  $\beta$  in the above-mentioned interval.

Encouraged by the very good agreement shown in Fig. 2 we have investigated the further potential uses of the theory. One of the objectives has been to extend the  $Z$  and  $\beta$  interval of the study reported in ref. [3]. Another objective has been to find out whether there exists a single quantity which can be used to characterize an emulsion stack and which consequently also can be useful for a comparison of different stacks. To do this we have collected experimental data for absorptance from a number of investigations published in the literature. These data have been compared with data calculated from Katz's theory. In the following some of the results will be discussed.

In Fig. 3 we show a comparison of integrated absorptance values. The measurements which are due to Söderström *et al.* [4] were made with a type 2 photometer in the same stack as used by us in ref. [3]. Thus we can use the

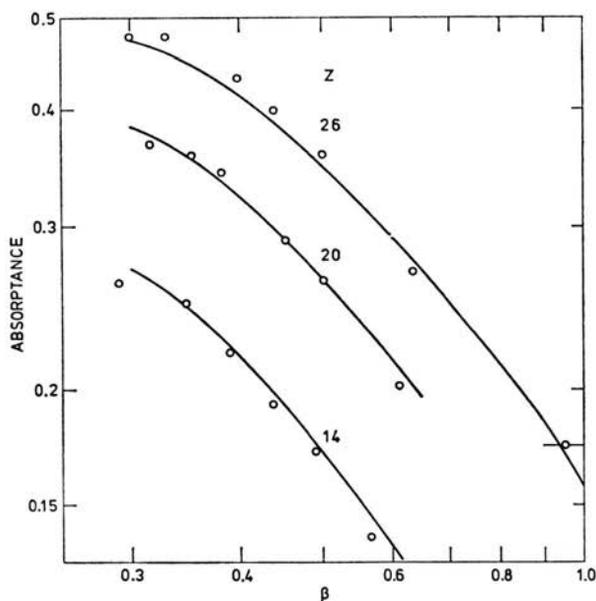


FIG. 3. — Integrated absorptance as a function of velocity for charge numbers  $Z = 14$ , 20 and 26. Theoretical relationships shown by solid curves, experimental points by circles. Velocity interval  $0.3 \leq \beta \leq 1.0$ .

same parameters of the theory as before and the comparison will show how well the theory can predict mean transmittance values, defined by

$$\bar{\tau}(y) = \frac{1}{\Delta y} \int_y^{y+\Delta y} \tau(y) dy, \quad (6)$$

where the integration is over the width  $\Delta y$  of the photometer slit. It should be pointed out that the beautiful agreement between experiment and theory in Fig. 3 has been obtained with no additional normalization. Note also that the theory yields reasonable absorptance values for the particles with  $\beta \approx 1$ , i.e. outside the previously investigated velocity interval.

From Figs. 2 and 3 we conclude that it is possible to relate different photometrical investigations to each other if they are made in the same stack. We now turn to the possibility of interrelating studies made in different stacks. Such a comparison would be possible if there exists some single parameter which describes the basic properties of an emulsion stack. The quantity  $E_0$  should be of such a character [2]. However, we prefer to use the ratio  $\sigma/E_0$ . The reason for this can be seen from the following. Inserting equations (4) and (2) into equation (3) we obtain

$$\tau_p(y) = \exp \left[ -\sigma n_0 \int [1 - \exp(-\bar{E}(t)/E_0)] dz \right]. \quad (7)$$

For low values of the energy dose, i.e.  $\bar{E}(t) \ll E_0$ , the above equation transforms to

$$\tau_p(y) \approx \exp \left[ -\frac{\sigma n_0}{E_0} \int \bar{E}(t) dz \right]. \quad (8)$$

Thus in this limit the values chosen for  $\sigma$ ,  $n_0$  and  $E_0$  become totally interdependent. We have seen this in ref. [3] where the fits based on different portions of the experimental data shown in Fig. 2 were found to yield values of  $\sigma$  and  $E_0$  which fluctuate by as much as 50 p. cent whereas the ratio  $\sigma/E_0$  was found to remain constant to better than 2 p. cent. From this we draw the conclusion that at the actual  $Z$  and  $\beta$  values, equation (8) is valid for a large portion of the experimental profile. Since both  $\sigma$  and  $E_0$  must in general be determined from a fit to experimental data, the above discussion shows that for technical reasons the use of  $\sigma/E_0$  yields more consistent results than  $E_0$ . In reality we have gathered all the detector-dependent constants in equation (8) to define an 'emulsion constant'

$$C = \sigma n_0 / E_0 \quad (9)$$

which we use to characterize a given emulsion stack.

We have tested the validity of the concept of a unique emulsion constant which characterizes a stack of emulsion, by bringing together experimental data obtained in two investigations [5, 6] made in an emulsion stack which

differs from the one used in the above-mentioned comparisons. The two studies employed both type 1 and type 2 photometers. In each case only one track was measured. The two tracks were located in different halves of the stack. The numerical values of  $C$  obtained from the optimization procedure were found to differ by less than 3 p. cent. Considering the above limitations this remarkable agreement may well be superficial. However, we can confidently state that the two determinations of  $C$  yield consistent results. Our conclusion then is that the emulsion constant  $C$  carries much of the information about an emulsion stack relevant to photometry of tracks of particles within the studied charge and velocity intervals.

The fundamental nature of  $C$  can be seen from a further study of equation (8). Consider a nuclear emulsion of thickness  $d$  uniformly exposed to a low dose  $E_u$ . This dose can be easily obtained as

$$E_u = \frac{D}{Cd}, \quad (10)$$

where we have introduced the optical density

$$D = \ln(\tau_0/\tau) \quad (11)$$

with  $\tau_0 = 1$ . The concept of the emulsion constant  $C$  is a direct consequence of Katz's original theory. The interesting result of the above discussion is that it points out the possibility to determine  $C$  in an unambiguous way.

We now turn to the question of extending the  $Z$  interval. From the theoretical point of view there should be no limitations to  $Z$  or  $\beta$  other than the one caused by the uncertainty about the very small energy transfers. However, for a real detector such as the nuclear emulsion we expect to meet difficulties both at high and at low doses. According to equation (1) this means that there should be corresponding limitations to  $Z$  and  $\beta$ .

Mean doses  $\bar{E} > E_0$  imply according to equation (2) an over-exposure which makes the response of the detector independent of  $Z$  and  $\beta$ . While this effect may well be correctly described by the theory, from the point of view of an experimentalist it is annoying. A still more serious consequence of the over-exposure is that individual silver grains tend to cluster together. On account of the growth of the silver grains during the development this effect begins to be of importance already at doses which are smaller than  $E_0$ . As a consequence of the clustering, equation (3) which is based on the concept of light scattering against randomly distributed individual silver grains cannot be expected to hold. This means that in the region  $\bar{E} \approx E_0$  there will be marked changes in the transport of light and thus in the combined response of the nuclear emulsion and the photometer. Local over-exposure along the path of individual  $\delta$ -rays obviously is a common effect also at distances  $t$  such that  $\bar{E}(t) < E_0$ . This has been discussed in the paper by Fowler *et al.* [7] who point out that in the region  $\bar{E} < E_0$  equation (3) should be appropriate in spite of the clustering occurring along individual  $\delta$ -rays. Clearly the cluste-

ring affects the physical interpretation of the cross-section  $\sigma$ . Katz *et al.* [8] call the interval  $\bar{E} > E_0$  the track width regime and the interval  $\bar{E} < E_0$  the grain count regime.

The lower limit to  $\bar{E}$  in emulsion is given by the spurious energy dose  $E_b$  which is simulated by the random fluctuation in the number of background grains.

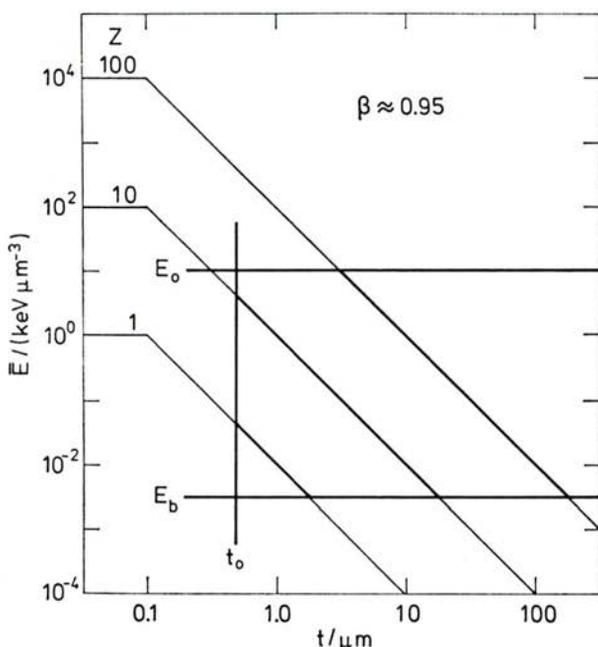


FIG. 4. — A qualitative sketch of the mean energy dose as a function of radial distance from track axis for relativistic ions with different charge number  $Z$ .

In addition to the limits  $E_0$  and  $E_b$  we have to consider a limit  $t_0$ , set by the spatial resolution which can be taken to be the same as the diameter of a developed grain, i.e. of the order of  $0.5 \mu\text{m}$ . These limits are illustrated in Fig. 4 which shows the qualitative dependence of  $\bar{E}$  on  $t$  for relativistic ions with different values of  $Z$ . The rectangular area confined between  $E_0$  and  $E_b$  and characterized by  $t > t_0$  can conveniently be called the  $\delta$ -ray regime. Fig. 4 clearly shows that for high values of  $Z$  the number of grains, i.e. the information content in the over-exposed track core is small compared with the number outside the core, i.e. in the  $\delta$ -rays. For low values of  $Z$  the opposite is true. Thus predictions based on equation (3) cannot be reliable as long as the track core is over-exposed and the total number of grains in the core is larger than in the  $\delta$ -rays.

According to these considerations we expect to find discrepancies between experiment and theory at charge numbers smaller than the ones previously

investigated. As seen by the comparison shown in Fig. 5 such discrepancies do exist. The comparison in Fig. 5 is made for integrated absorbance values for  $Z = 6$  at different velocities. Experimental points are taken from the study by Jensen [6]. The solid line is the prediction from theory. The emulsion constant needed for the calculation was determined from a fit to experimental data in the charge number interval  $14 \leq Z \leq 26$  where the theory is known to be valid. Unfortunately, from lack of experimental data, we have not been able to extend the comparison to other charge numbers  $Z < 14$ . Thus we can only state that the theory breaks down somewhere in the charge interval  $6 < Z < 14$ . From the above discussion it should be clear that the deterioration of the quality of the theoretical predictions must be gradual.

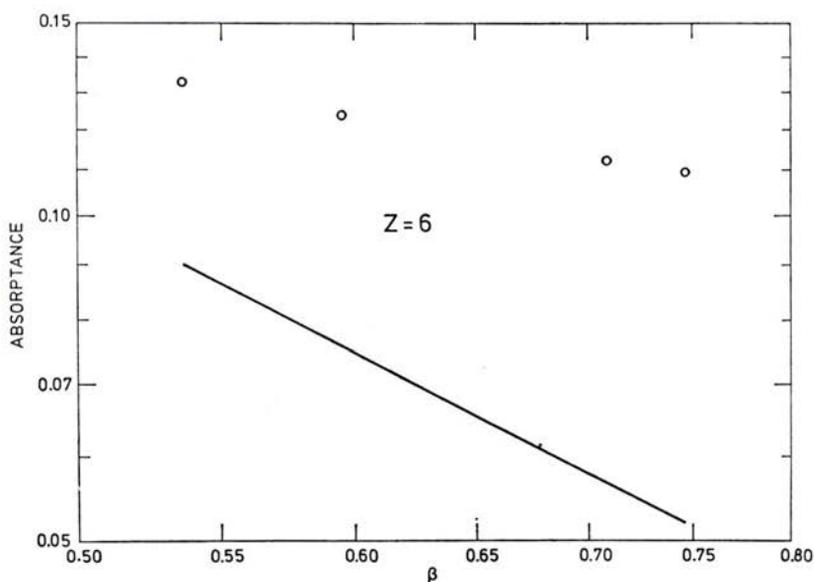


FIG. 5. — Integrated absorbance as a function of velocity for charge number  $Z = 6$ . Theoretical relationships shown by solid line, experimental points by circles. Velocity interval  $0.5 \leq \beta \leq 0.8$ .

For an investigation of the behaviour of the theory at  $Z > 26$  we have used the experimental data by Fowler et al. [7]. These data and our theoretical predictions are shown in Fig. 6. The emulsion constant needed for the calculation was determined from a fit to experimental data at  $Z = 26$ . We have assumed that the charge groups for which optical density profiles have been given by Fowler et al. [7] have the average charge numbers  $\langle Z \rangle$  and velocities  $\beta$  according to the Table. As can be seen from Fig. 6 the agreement between theory and experiment is amazingly good, the existing discrepancies most probably being due to incorrect estimates of the average charge values  $\langle Z \rangle$ . It should be pointed out that in our calculation no normalization of

the theoretical data for  $Z > 26$  has been undertaken, in contrast to the method employed by Fowler *et al.* [7].

$Z$ group	$\langle Z \rangle$	$\beta$
26	26	0.94
35 to 50	42	0.93
40 to 60	50	0.92
55 to 83	70	0.91
102	100	0.90

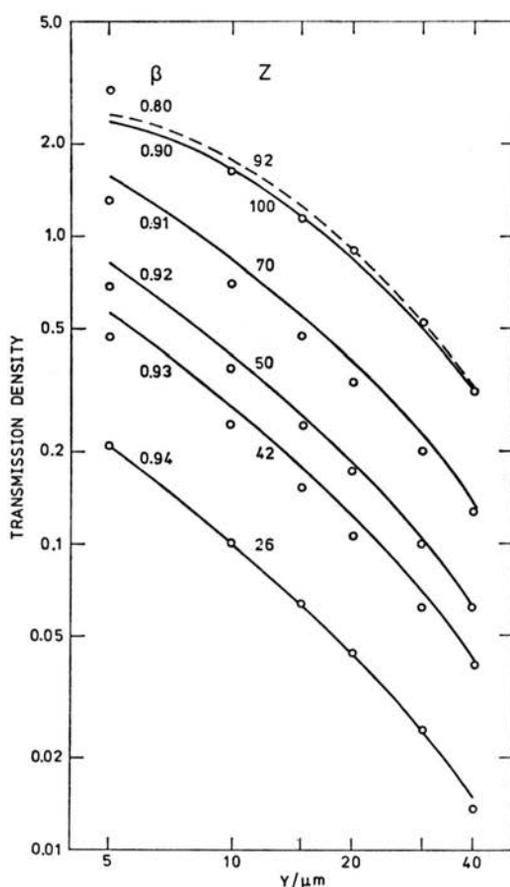


FIG. 6. — Optical density as a function of lateral distance from track axis for charge numbers  $Z \geq 26$ . Theoretical relationships given by solid lines, experimental points (due to Fowler *et al.* [7]) by circles. Velocity interval  $0.80 \leq \beta \leq 0.94$ .

Our calculated profile for  $Z = 100$  and  $\beta = 0.90$  matches the experimental data for the ultra heavy particle with an assigned charge number of  $Z \approx 102$  very well, except for the point at the lateral distance of  $5 \mu\text{m}$ . Thus our calculations can be taken as an independent check of the general correctness of the charge determination used by Fowler *et al.* [7]. However, it must be pointed out that the level of the density profile depends very strongly on the velocity, the form of the profile unfortunately being almost independent of  $\beta$ . This means that different sets of  $Z$  and  $\beta$  can describe the same experimental profile with good accuracy. To demonstrate this we show in Fig. 6 also a calculated profile for  $Z = 92$  and  $\beta = 0.80$ . This profile is seen to follow very closely the profile for  $Z = 100$  and  $\beta = 0.90$ . According to the energy spectrum shown by Fowler *et al.* [7] it seems quite possible that in their experiment an ion with  $Z = 92$  can have  $\beta = 0.8$  at the detector level. This means that for a positive identification of transuranic elements in the cosmic radiation by the method employed by Fowler *et al.* [7] an extremely careful determination of particle velocity is needed.

From this part of the investigation we draw the conclusion that Katz's theory is capable of describing the structure of tracks of fast ions in the entire charge interval  $Z \geq 14$ .

Finally we wish to make some comments about the limitations of our study. These points could be of some value for people who are going to apply Katz's theory of track formation for detectors which have a response different from that of nuclear emulsion.

1. The sensitive element of the nuclear emulsion, i.e. the AgBr crystal has a much simpler structure than the sensitive element of most other detectors, such as living cells or even dielectrics.

2. The above results are valid for a situation where the entire energy dose is deposited within a very short interval of time. It is known that the *repair* of nuclear emulsion, i.e. the fading of the latent image, is extremely slow. From this one could infer that the same end effect as observed by us would result if the appropriate energy dose was delivered in small steps during a long interval of time. However, this problem has not been dealt with in the present study.

3. Our investigation yields no information about the behaviour of Katz's theory at distances smaller than about  $0.5 \mu\text{m}$  from the track axis.

4. We have not studied the behaviour of the theory in a systematic way for charge numbers  $Z < 14$  and particle velocities outside the interval  $0.3 \leq \beta \leq 0.8$ .

5. Within the above limitations we find Katz's theory to be very well suited for a description of the structure of tracks of ionizing particles in nuclear emulsion.

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